

## CHAPTER 4

# APPLICATIONS OF ENERGY BALANCES IN THE ABSENCE OF CHEMICAL REACTIONS

- 1 Simplifications of the General Energy Balance
- 2 The Strategy for Solving Energy Balance Problems
- 3 Application of the Energy Balance to Closed Systems
- 4 Application of the Energy Balance to Open Systems

## 1 Simplifications of the General Energy Balance

simplify the general energy balance developed in Chapter 2. We reproduce it here using concise symbols as in Equation (6) with “inside” and “flow” appended to refresh your memory about the system. “Inside” refers to the system inside the system boundary while “flow” refers to streams that cross the system boundary.

$$\begin{aligned}\Delta E_{\text{inside}} &= \Delta(U + PE + KE)_{\text{inside}} = Q + W - \Delta(H + PE + KE)_{\text{flow}} \\ &= Q + W - \Delta[m(\hat{H} + \hat{PE} + \hat{KE})]\end{aligned}\quad (1)$$

The three most commonly used applications of Equation (24.1) are to:

1. closed systems
2. open systems with heat transfer into and out of the process
3. open, steady-state flow systems.

What follows is an analysis of each of these cases along with some physical examples:

1. **Closed system.** For closed or **batch systems**, no mass flow occurs in or out of the system ( $m_1 = m_2 = 0$ ) so that:  $\Delta E = Q + W$  (2)

If there is no accumulation ( $\Delta E = 0$ ),

$$Q = -W \quad (3)$$

An example is the initial heating to expand a hot air balloon.

2. **Open system with heat transfer.** Many open, steady-state processes in the chemical processing industries exist, processes that are dominated by heat transfer,  $Q$ . For these cases,  $\Delta E$ ,  $W$ ,  $\Delta PE$ , and  $\Delta KE$  can be neglected because they are small compared to  $Q$  and  $\Delta H$  or are zero. For these cases, Equation (1) reduces to

$$Q = \Delta[m\hat{H}] = \Delta H \quad (4)$$

Equation (4) can be applied to heat exchangers (i.e., devices for transferring heat from a high temperature fluid to a lower temperature fluid) and distillation columns.

If no heat transfer takes place between the system and the surroundings, Equation (4) reduces to

$$\Delta H = 0 \quad (5)$$

Equation (5) is sometimes called the “**enthalpy balance**,” and is used to model the mixing of two fluids at different temperatures.

3. **Open, steady-state flow system.** For steady-state ( $\Delta E = 0$ ) flow systems that do not involve significant heat transfer ( $Q = 0$ ), Equation (24.1) reduces to

$$W = \Delta PE + \Delta KE \quad (6)$$

You can use Equation (6) to size pumps or calculate the pressure drop through a piping network.

Some special process names associated with energy balance problems that we have mentioned in previous chapters are worth remembering:

- a. *Isothermal* ( $dT = 0$ ): constant temperature process
- b. *Isobaric* ( $dp = 0$ ): constant pressure process
- c. *Isometric or isochoric* ( $dV = 0$ ): constant volume process
- d. *Adiabatic* ( $Q = 0$ ): no heat interchange. If we inquire as to the circumstances under which a process can be called adiabatic, one of the following is most likely:
  - (1) the system is insulated.
  - (2)  $Q$  is very small in relation to the other terms in the energy equation and may be neglected.
  - (3) the process takes place so fast that there is no time for heat to be transferred.

**EXAMPLE 24.1 Simplification of the General Energy Balance**

Figure E24.1 illustrates a process in which several segments are distinguished by numbered boundaries. List the simplifying assumptions to make in Equation (1) for the following segments: 1 to 5, 4 to 5, 3 to 4, 3 to 5, and 1 to 3.

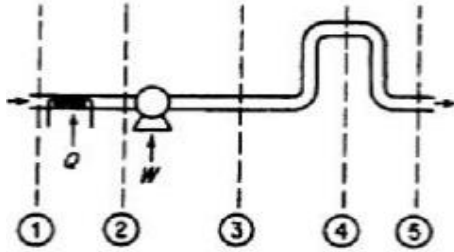


Figure E24.1

$$4 \text{ to } 5: \bullet \hat{Q} = \hat{W} = 0$$

$$\bullet \Delta \hat{K}E \cong 0$$

$$\bullet \Delta \hat{E} = 0$$

$$\text{Result: } \Delta \hat{H} = -\Delta \hat{P}E$$

$$3 \text{ to } 4: \bullet \hat{Q} = \hat{W} = 0$$

$$\bullet \Delta \hat{K}E \cong 0$$

$$\bullet \Delta \hat{E} = 0$$

$$\text{Result: } \Delta \hat{H} = -\Delta \hat{P}E$$

Here is the analysis for each segment

$$1 \text{ to } 5: \bullet \Delta \hat{P}E = 0 \text{ (no level change)}$$

$$\bullet \text{Probably } \Delta \hat{K}E = 0$$

$$\bullet \Delta \hat{E} = 0 \text{ (process appears to be steady state)}$$

$$\text{Result: } \hat{Q} + \hat{W} = \Delta \hat{H}$$

$$3 \text{ to } 5: \bullet \hat{Q} = \hat{W} = 0$$

$$\bullet \Delta \hat{P}E = 0$$

$$\bullet \Delta \hat{K}E \cong 0$$

$$\bullet \Delta \hat{E} = 0$$

$$\text{Result: } \Delta \hat{H} = 0$$

$$1 \text{ to } 3: \bullet \Delta \hat{P}E = 0$$

$$\bullet \Delta \hat{K}E \text{ may be approximately } 0$$

$$\bullet \Delta \hat{E} = 0$$

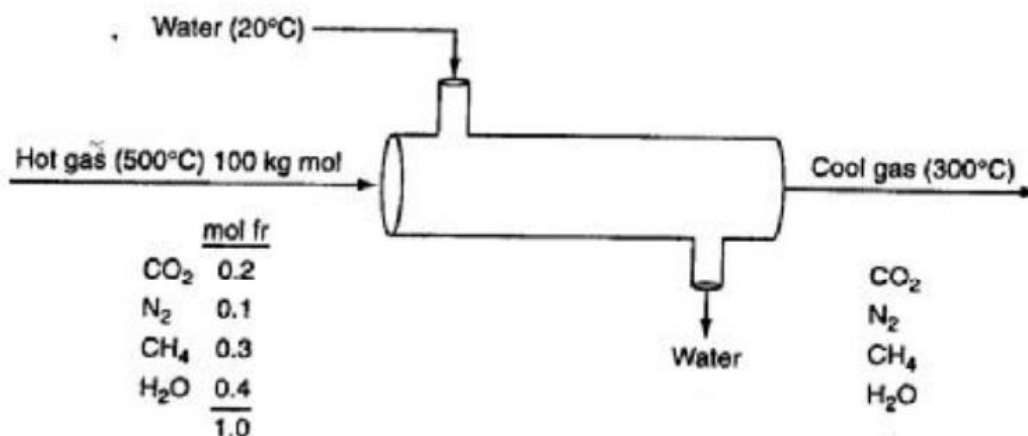
$$\text{Result: } \hat{Q} + \hat{W} = \Delta \hat{H}$$

**2 The Strategy for Solving Energy Balance Problems**

1. Choose the system, identify its boundary, and then decide whether the system is open or closed.
2. Decide if the system is steady state or unsteady state. Write the decision down.
3. Write down the general energy balance, Equation (1) or (.7) for the system (one for each system if you pick multiple systems) along with the other equations.
4. Simplify the general energy balance as much as possible by using information in the problem statement and reasonable assumptions based on your understanding of the process.
5. Carry out a degree-of-freedom analysis that includes the energy balance as one equation (that may include more than one additional variable) along with the material balances.
6. Choose a reference state for your calculations, usually the specification of temperature and pressure, but other variables may be substituted.
7. Based on the reference state, get any needed physical property data ( $T$ ,  $p$ ,  $\hat{V}$ ,  $\hat{U}$ ,  $\hat{H}$ ) and add the values of the data to the sketch of the process. Be sure to include phase changes.
8. Solve the energy balance alone or in conjunction with the material balances.

### EXAMPLE 24.2 Degree-of-Freedom Analysis Including an Energy Balance

Figure E24.2 shows a hot gas stream at 500°C being cooled to 300°C by transferring heat to the liquid water that enters at 20°C. You want to find the water flow rate per 100 kg mol of entering hot gas. Carry out a degree-of-freedom analysis for this problem to determine if you can solve it based on the information given above and in Figure E24.2.



#### Solution

Because  $Q = W = 0$  and  $\widehat{PE} = \widehat{KE} = 0$  in each stream, with the system being open and steady state, the energy balance reduces to  $\Delta H = 0$ . Because the problem has 4 components, you can write four independent material balances. Suppose we take the viewpoint that the flow of each component (stream flow) is a variable. Then, considering all of the variables, the problem involves nine stream flow variables (some of which are known, of course), and three temperatures. As for the stream pressures, we have to assume they are specified; implicitly the water will be at its vapor pressure.

Based on the above preliminary analysis, you can form a table to carry out the degree-of-freedom analysis.

*Number of variables involved:*

Material

Hot gas:	4 component flows, $T$ , and $p$	6
Cool gas:	4 component flows, $T$ , and $p$	6
Water in:	1 component flow, $T$ , and $p$	3
Water out:	1 component flow, $T$ , and $p$	3

<b>Energy:</b>	
$Q$ and $W$	2
$H$ , $PE$ , and $KE$ associated with each stream flow	<u>12</u>
<b>Total</b>	<u>32</u>
<b>Number of equations and specifications:</b>	
<b>Specified values</b>	
Hot gas: $T$ , $p$ , and 4 component flows	6
Cool gas: $T$ and $p$	2
Water in: $T$ and $p$	2
Water out: $T$ and $p$	2
<b>Specified in the energy balance:</b>	
$Q$ and $W$	2
$PE$ and $KE$ (in 4 streams)	8
Material balances: 4 species balances plus water	5
Energy balance:	1
$H$ in each stream is a function of the specified $T$ and $p$	<u>4</u>
	<u>32</u>
<b>Degrees of freedom</b>	0

### 3 Applications of the Energy Balance to Closed Systems

#### EXAMPLE 24.3 Application of the Energy Balance

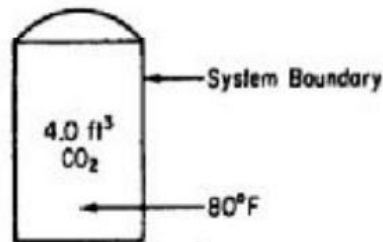
Ten pounds of  $\text{CO}_2$  at room temperature ( $80^\circ\text{F}$ ) are stored in a fire extinguisher having a volume of  $4.0 \text{ ft}^3$ . How much heat must be removed from the extinguisher so that 40% of the  $\text{CO}_2$  becomes liquid?

#### Solution

This problem involves a closed, unsteady-state system (Figure E24.3) without reaction. You can use the  $\text{CO}_2$  chart in Appendix J to get the necessary property values.

**Steps 1, 2, 3, and 4**

The specific volume of the  $\text{CO}_2$  is  $4.0/10 = 0.40 \text{ ft}^3/\text{lb}$ , hence you can find on the  $\text{CO}_2$  chart at  $\hat{V} = 0.40$  and  $T = 80^\circ\text{F}$  that  $\text{CO}_2$  is a gas at the start of the process. The reference state for the  $\text{CO}_2$  chart is  $-40^\circ\text{F}$ , saturated liquid. The pressure is 300 psia and  $\Delta\hat{H} = 160 \text{ Btu/lb}$ .

**Figure E24.3****Step 5**

Basis: 10 lb  $\text{CO}_2$

**Steps 6 and 7**

The material balance is trivial—the mass in the system is constant at 10 lb. In the energy balance

$$\Delta E = Q + W$$

$W$  is zero because the volume of the system is fixed, hence with  $\Delta KE = \Delta PE = 0$  inside the system

$$Q = \Delta U = \Delta H - \Delta(pV)$$

You cannot obtain values of  $\Delta\hat{U}$  from the  $\text{CO}_2$  chart, only values of  $\Delta\hat{H}$ , by following the constant-volume line of  $0.40 \text{ ft}^3/\text{lb}$  to the spot where the quality is 0.6. Hence the final state is fixed, and all the final properties can be identified, namely

$$\Delta\hat{H}_{\text{final}} = 81 \text{ Btu/lb}$$

$$p_{\text{final}} = 140 \text{ psia}$$

You can conclude that the degrees of freedom for the problem are zero, with one equation in one unknown to be solved for  $Q$ .

**Steps 7, 8, and 9**



$$Q = \left\{ (81 - 160) - \left[ \frac{(140)(144)(0.40)}{778.2} - \frac{(300)(144)(0.40)}{778.2} \right] \right\} 10$$

$$= -672 \text{ Btu (heat is removed)}$$

### EXAMPLE 24.4 Application of the Energy Balance to Plasma Etching

Argon gas in an insulated plasma deposition chamber with a volume of 2 L is to be heated by an electric resistance heater. Initially the gas, which can be treated as an ideal gas, is at 1.5 Pa and 300 K. The 1000-ohm heater draws current at 40 V for 5 minutes (i.e., 480 J of work is done on the system by its surroundings). What is the final gas temperature and pressure in the chamber? The mass of the heater is 12 g and its heat capacity is 0.35 J/(g)(K). Assume that the heat transfer through the walls of the chamber from the gas at this low pressure and in the short time period considered is negligible.

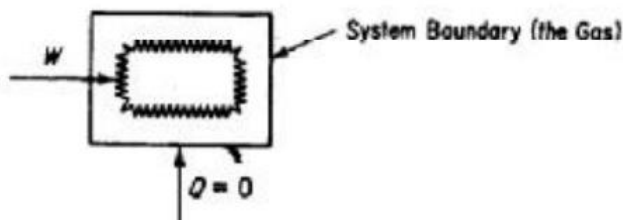


Figure E24.4

#### Solution

No reaction occurs. The fact that the heater coil is "heated" inside the system does not mean that heat transfer takes place to the selected system from the surroundings. The system does not exchange mass with the surroundings, and is unsteady state.

#### Steps 1, 2, 3, and 4

Pick the system as the gas plus the heater, as shown in Figure E24.4. Because of the assumption about the heat transfer,  $Q = 0$ .  $W$  is given as +480 J (work done on the system) in 5 minutes.

#### Step 5

Basis: 5 minutes

#### Steps 6 and 7

The general energy balance (with  $\Delta PE = \Delta KE = 0$  inside the system) is

$$\Delta E = \Delta U = Q + W$$

and reduces to  $\Delta U = 480 \text{ J}$

The assumption that the gas is ideal simplifies the solution because for an ideal gas

$$pV = nRT$$

Initially we know  $p$ ,  $V$ , and  $T$ , and thus can calculate the amount of the gas:

$$\begin{aligned} n &= \frac{pV}{RT} = \frac{1.5 \text{ Pa} \left| \frac{2 \text{ L} \left| 10^{-3} \text{ m}^3 \right. \right.}{1 \text{ L}} \left| \frac{1 \text{ (g mol)(K)}}{8.314 \text{ (Pa)(m}^3)} \right. \right.}{300 \text{ K}} \\ &= 1.203 \times 10^{-6} \text{ g mol} \end{aligned}$$

You are given the heater mass and a heat capacity of  $C_v = 0.35 \text{ J/(g)(K)}$ , and the  $C_p$  of the gas can be calculated. Since  $C_p = \frac{5}{2}R$  (see Table 23.1), then

$$C_v = C_p - R = \frac{5}{2}R - R = \frac{3}{2}R$$

You have to pick a reference temperature for the calculations. The most convenient reference state is 300K. Then  $\Delta U$  for the gas and the heater can be calculated, assuming both the heater and the gas end up at the same temperature:

*Gas:*

$$\Delta U_g = 1.203 \times 10^{-6} \int_{300}^T C_v dT' = 1.203 \times 10^{-6} \left( \frac{3}{2}R \right) (T - 300)$$

*Heater:*

$$\Delta U_h = 12g \left( \frac{0.35 \text{ J}}{\text{(g)(K)}} \right) (T - 300)$$

The mass balance is trivial—the mass in the chamber does not change. The unknown is  $T$ , and one equation is involved, the energy balance, so that the degrees of freedom are zero.

### Steps 8 and 9

Because  $\Delta U = 480 \text{ J}$ , you can calculate  $T$  from the energy balance

$$\Delta U = 480 \text{ J} = \frac{\text{heater}}{(12)(0.35)(T - 300)} + \frac{\text{gas}}{(2.302 \times 10^{-6}) \left( \frac{3}{2} \right) (8.314)(T - 300)}$$



$$T = 414 \text{ K}$$

The final pressure is

$$\frac{p_2 V_2}{p_1 V_1} = \frac{n_2 R T_2}{n_1 R T_1}$$

or

$$p_2 = p_1 \left( \frac{T_2}{T_1} \right) = 1.5 \left( \frac{414}{300} \right) = 2.07 \text{ Pa}$$

## 4 Application of the Energy Balance to Open Systems

As a chemical engineer you will encounter a wide variety of processes in any plant. Most of the processes will be modeled as open (flow), steady-state systems. Operations such as the transport of solids and liquids, heat exchange, evaporation, and so on involve open systems. You will find that even in complex plants the essentials of analysis reduce to the review of the material and energy flows, as illustrated in Figure 24.3. At each stage there can be loss of material and energy, losses that can go into the environment, losses that engineers try to reduce economically.

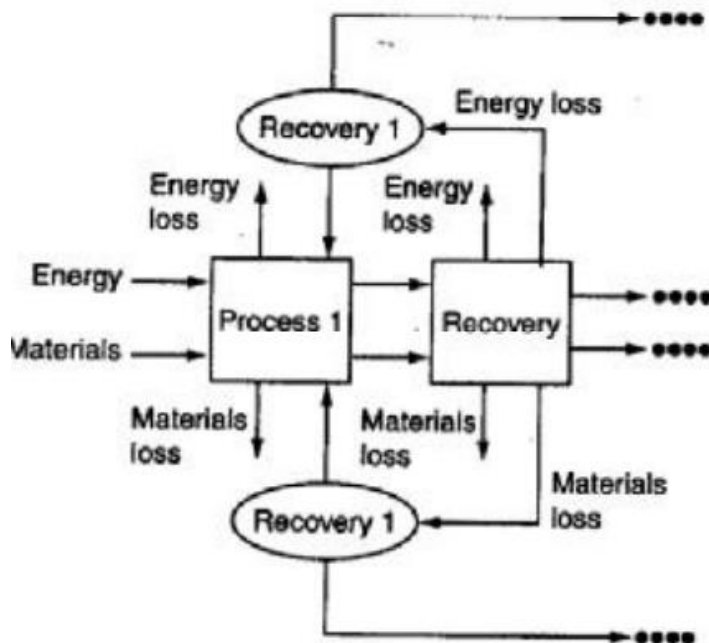


Figure 24.3 General energy and materials flow in a complex plant.

### EXAMPLE 24.6 Application of the Energy Balance to Pumping Water

Water is being pumped from the bottom of a well 15 ft deep at the rate of 200 gal/hr into a vented storage tank to maintain a level of water in a tank 165 ft above the ground. To prevent freezing in the winter a small heater puts 30,000 Btu/hr into the water during its transfer from the well to the storage tank. Heat is lost from the whole system at the constant rate of 25,000 Btu/hr. What is the temperature of the water as it enters the storage tank, assuming that the well water is at 35°F? A 2-hp pump is being used to pump the water. About 55% of the rated horsepower goes into the work of pumping and the rest is dissipated as heat to the atmosphere.

#### Solution

##### Steps 1, 2, 3, and 4

Let the system (assume an open, steady-state system) consist of the well inlet, the piping, the pump, and the outlet at the storage tank. See Figure E24.6.

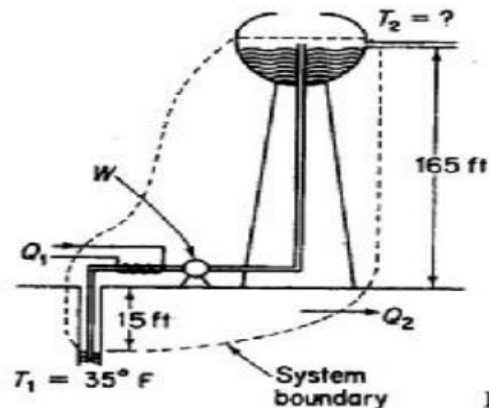


Figure E24.6

##### Step 5

Basis: 1 hr of operation.

##### Steps 6 and 7

The material balance is 200 gal of water entering and 200 gal leaving in an hour. The energy balance is

$$\Delta E = Q + W - \Delta[(\hat{H} + \hat{K}E + \hat{P}E)m]$$

You can simplify the energy balance as follows:

1. The process is in the steady state, so that  $\Delta E = 0$ .
2.  $m_1 = m_2 = m$ .
3.  $\Delta KE \approx 0$  because we will assume that  $v_1 = v_2 \approx 0$ .

Then

$$0 = Q + W - \Delta[(\hat{H} + \hat{P}E)m]$$

**Steps 8 and 9**

The total mass of water pumped is  $\frac{200 \text{ gal}}{\text{hr}} \left| \frac{8.33 \text{ lb}}{1 \text{ gal}} \right| = 1666 \text{ lb}$

The potential energy change is  $\Delta PE = m\Delta\widehat{PE} = mg\Delta h$

$$= \frac{1666 \text{ lb}_m \left| \frac{32.2 \text{ ft}}{\text{s}^2} \right| \left| \frac{180 \text{ ft}}{32.2 (\text{ft})(\text{lb}_m)} \right| \left| \frac{(\text{s}^2)(\text{lb}_f)}{778 (\text{ft})(\text{lb}_f)} \right|}{1} = 385.4 \text{ Btu}$$

The heat lost by the system is 25,000 Btu while the heater puts 30,000 Btu into the system; hence the net heat exchange is  $Q = 30,000 - 25,000 = 5000 \text{ Btu}$

The rate of work being done on the water by the pump is

$$\dot{W} = \frac{2 \text{ hp} \left| \frac{0.55}{(\text{min})(\text{hp})} \right| \left| \frac{33,000 (\text{ft})(\text{lb}_f)}{60 \text{ min}} \right| \left| \frac{\text{Btu}}{778 (\text{ft})(\text{lb}_f)} \right|}{\text{hr}} = 2800 \text{ Btu/hr}$$

hence  $W = 2800 \text{ Btu}$

$\Delta H$  can be calculated from:  $Q + W = \Delta H + \Delta PE$

$$5000 + 2800 = \Delta H + 385 \quad \Delta H = 7415 \text{ Btu}$$

Because the temperature range considered is small, the heat capacity of liquid water may be assumed to be constant and equal to 1.0 Btu/(lb)(°F) for the problem.

Thus,

$$\Delta H = m\Delta\widehat{H} = m \int_{T=35^\circ\text{F}}^{T_2} C_p dT = mC_p(T_2 - 35)$$

$$7415 = \Delta H = mC_p \Delta T = 1666(1.0)(T_2 - 35)$$

$$\Delta T \cong 4.5^\circ\text{F} \text{ temperature rise, hence, } T_2 = 39.5^\circ\text{F.}$$

### EXAMPLE 24.7 Application of the Energy Balance to Heating a Biomass

Steam at 250°C saturated (that is used to heat a fermentation broth) enters the steam chest of a fermentor. The steam chest is segregated from the biomass in the fermentor. Assume that the steam is completely condensed in the steam chest. The rate of the heat loss from the fermentor to the surroundings is 1.5 kJ/s. The material to be heated is placed in the fermentor at 20°C and at the end of the heating it is at 100°C. If the charge consists of 150 kg of material with an average heat capacity of  $C_p = 3.26 \text{ J/(g)(K)}$ , how many kilograms of steam are needed per kilogram of charge? The charge remains in the vessel for 1 hr.

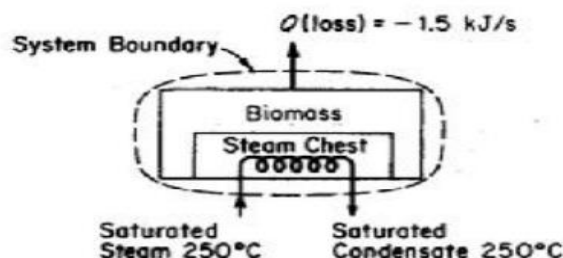


Figure E24.7a

#### Solution

##### Steps 1, 2, 3, and 4

Figure E24.7a defines the system and shows the known conditions. If the system is the biomass plus the steam chest, the process is open but not a steady state one because the temperature of the biomass increases. Zero degrees of freedom exist so that the problem has a solution.

##### Step 5

Basis: 1 hr of operation (150 kg of charge)

##### Steps 6, 7, and 8

The steam is the only material entering and leaving the system, and  $m_1 = m_2$  of the steam, hence the material balance is simple. The energy balance is

$$\Delta E = Q + W - \Delta[(\hat{H} + \hat{KE} + \hat{PE})m] \quad (a)$$

Let us simplify the energy balance

1. The process is not in the steady state, so  $\Delta E \neq 0$ .
2. We can safely assume that  $\Delta KE = 0$  and  $\Delta PE = 0$  inside the system.
3.  $W = 0$ .
4.  $\Delta KE$  and  $\Delta PE$  of the entering and exiting material are zero.

Consequently, Equation (a) becomes

$$\Delta E = \Delta U = Q - \Delta[(\hat{H})m] \quad (b)$$

(a)  $\Delta U = \Delta H - \Delta(pV) = \Delta H = m_{\text{biomass}} C_{p, \text{biomass}} (373 - 293)\text{K}$  [because we know that  $\Delta(pV)$  for the liquid or solid charge is negligible]. Thus

$$\Delta U = \Delta H_{\text{biomass}} = \frac{150 \text{ kg}}{1} \left| \frac{3.26 \text{ kJ}}{(\text{kg})(\text{K})} \right| \frac{(373 - 293)\text{K}}{1} = 39,120 \text{ kJ}$$

(b) The heat loss is given as  $Q = -1.50\text{kJ/s}$

$$\frac{-1.50 \text{ kJ}}{\text{s}} \left| \frac{3600 \text{ s}}{1 \text{ hr}} \right| \frac{1 \text{ hr}}{1} = -5400 \text{ kJ}$$

(c) The specific enthalpy change for the steam (if the changes consist only of the  $\Delta \hat{H}$  of condensation) can be determined from the steam tables. The  $\Delta \hat{H}_{\text{vap}}$  of saturated steam at  $250^\circ\text{C}$  is  $1701 \text{ kJ/kg}$ , so that

$$\Delta \hat{H}_{\text{steam}} = -1701 \text{ kJ/kg}$$

Introduction of all these values into Equation (b) gives

$$39,120 \text{ kJ} = - \left( -1701 \frac{\text{kJ}}{\text{kg steam}} \right) (m_{\text{steam}} \text{ kg}) - 5400 \text{ kJ} \quad (\text{c})$$

from which the kilograms of steam per hour,  $m_{\text{steam}}$ , can be calculated as

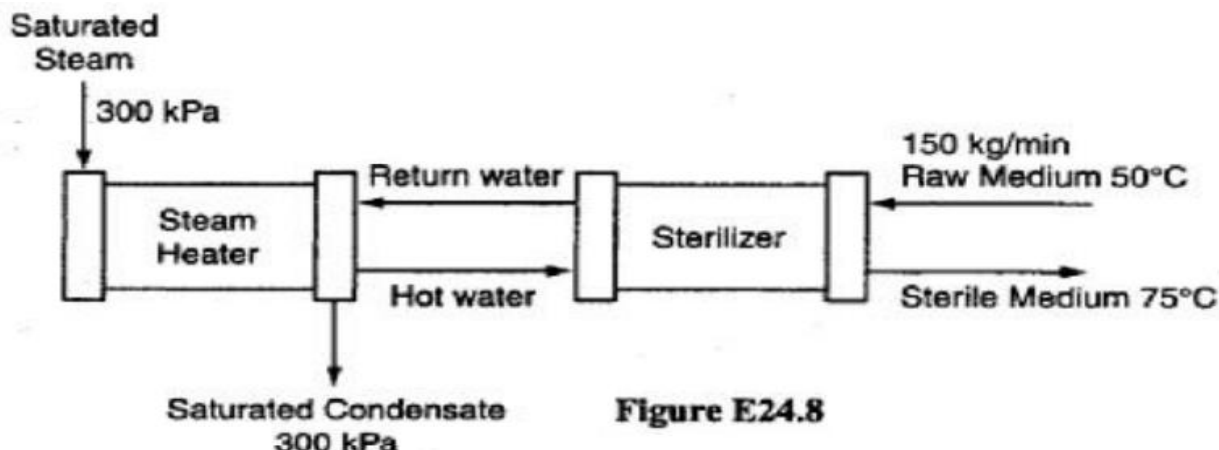
$$m_{\text{steam}} = \frac{44,520 \text{ kJ}}{1701 \text{ kJ}} \left| \frac{1 \text{ kg steam}}{1} \right| = 26.17 \text{ kg steam}$$

Thus

$$\frac{26.17 \text{ kg steam}}{150 \text{ kg charge}} = 0.174 \frac{\text{kg steam}}{\text{kg charge}}$$

### EXAMPLE 24.8 Sterilization of a Fermentation Medium

For a biological media (or medical instruments) the primary objective of sterilization (pasteurization) is to destroy undesirable microorganisms. At the same time, you want to avoid nutrient degradation and minimize costs. Figure E24.8 shows a process in which a biological media passes through a sterilizer heated by hot water. The return hot water is heated by condensing steam. The total heat loss from the process is  $1.63 \text{ kW}$ . Figure E24.8 shows the conditions for the process flows. Calculate the flow rate of the steam entering the steam heater.

**Solution**

Both material and energy balances are involved in the solution of this problem.

**Steps 1–4**

The basic data are in Figure E24.8. From the steam tables (assuming that the biomedica has the properties of water), you can obtain the necessary enthalpies:

	$\Delta \hat{H}$ (kJ/kg)
Water, 50°C	207.5
Water, 75°C	310.3
Steam, saturated, 300 kPa	2724.9
Water, saturated, 300 kPa	561.2

**Step 5** Basis: 150 kg biomedica (1 min)

**Steps 6 and 7** the system is open and steady state.

The material balances are quite simple: what goes in the sterilizer comes out, namely 150 kg of biomedica, and what goes in the steam heater comes out, but we do not know the value. Let's call it  $m$  kg.

The degrees of freedom are zero because you have only one unknown quantity,  $m$ , and you can write down one energy balance as follows. Because the system is in the steady state with no work,  $KE$ , or  $PE$  involved, the general energy balance reduces to

$$Q = \Delta H = H_{\text{out}} - H_{\text{in}}$$

$$\frac{1.63 \text{ kW}}{(1 \text{ W})(\text{s})} \left| \frac{1 \text{ J}}{\text{s}} \right| \frac{60 \text{ s}}{1 \text{ min}} \left| \frac{1 \text{ min}}{1 \text{ min}} \right| = \frac{150 \text{ kg } F}{\text{kg } F} \left| \frac{310.3 \text{ kJ}}{\text{kg } F} \right| + \frac{m \text{ kg water}}{\text{kg water}} \left| \frac{561.2 \text{ kJ}}{\text{kg water}} \right|$$

$$- \frac{150 \text{ kg } F}{\text{kg } F} \left| \frac{207.5 \text{ kJ}}{\text{kg } F} \right| - \frac{m \text{ kg steam}}{\text{kg steam}} \left| \frac{2724.9 \text{ kJ}}{\text{kg steam}} \right|$$

$$m = 7.08 \text{ kg of steam per minute}$$



### EXAMPLE 24.9 Use of Combined Material and Energy Balances to Solve a Distillation Problem

A feed of a mixture of benzene (Bz) and toluene (Tol) is separated as shown in Figure E24.9a. Calculate the values of the distillate ( $D$ ), the bottoms ( $B$ ), the heat duty (heat removed by the water) in the condenser ( $Q_C$ ), and the heat duty (input) to the reboiler ( $Q_R$ ). The recycle ratio  $R/D$  is 4.0.

#### Solution

The entering and exit streams  $F$ ,  $D$ , and  $B$  are all liquids (presumably saturated liquids). The system is open and steady state ( $\Delta E = 0$ ).

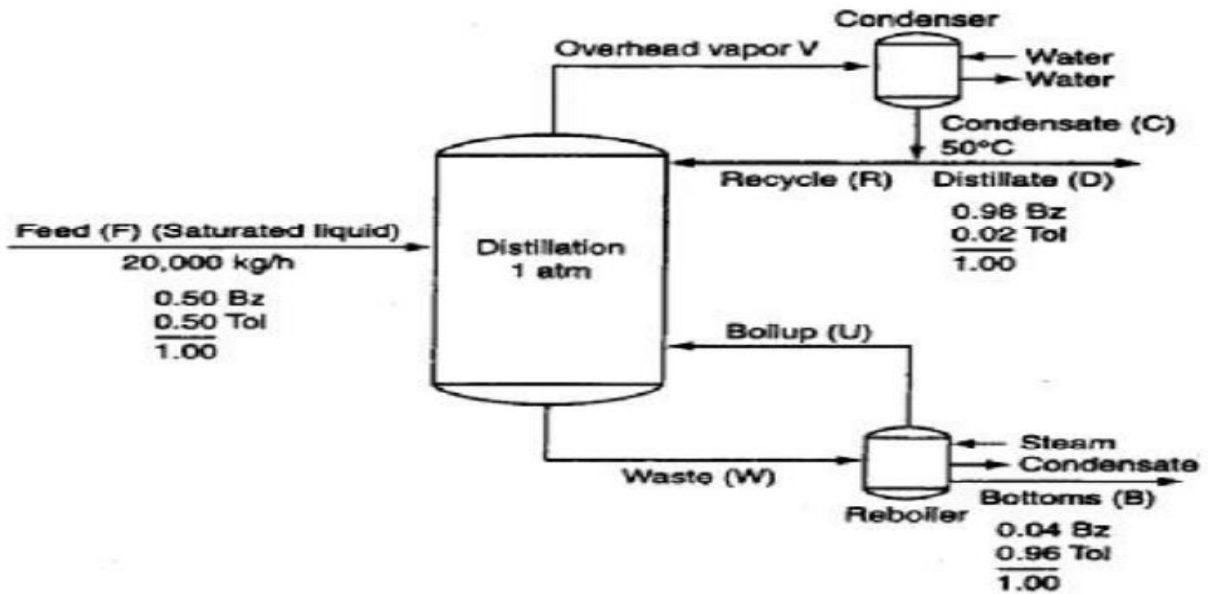


Figure E24.9a

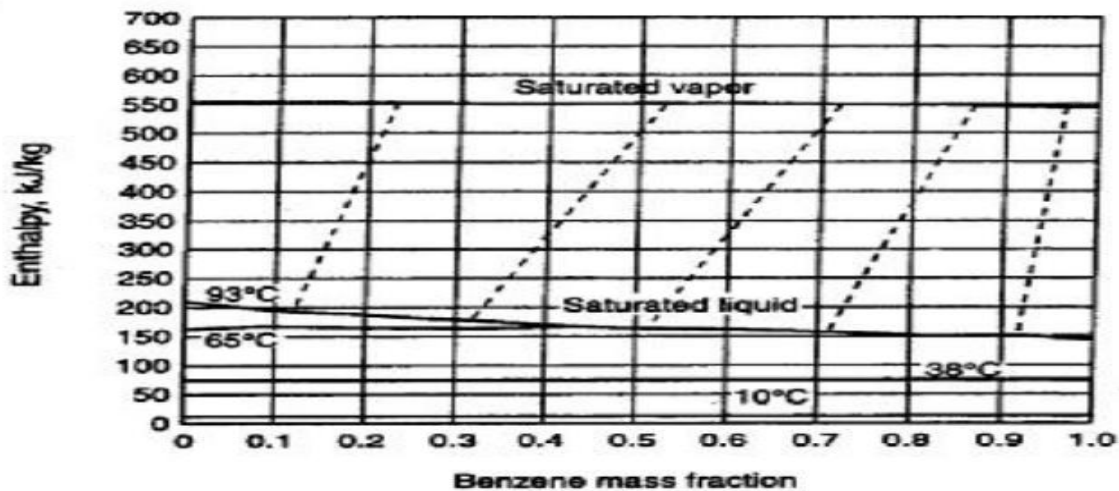


Figure E24.9b The ends of the dashed lines show the equilibrium compositions between the liquid and vapor.

Basis: 1 hour = 20,000 kg of feed

Steps 6 and 7 degree-of-freedom analysis is

Variables:	$B$ and $D$	2
Material balances:	$Bz$ and $Tol$	<u>2</u>
Degrees of freedom		0

Steps 8 and 9

You can write three material balances, only two of which are independent

$$\text{Total} \quad 20,000 \quad = D + B$$

$$\text{Bz:} \quad (0.50)(20,000) = 0.98D + 0.04B$$

$$\text{Tol:} \quad (0.50)(20,000) = 0.02D + 0.96B$$

Solve using the last two of the three equations to get

$$D = 9.8 \times 10^3 \text{ kg} \quad B = 1.02 \times 10^4 \text{ kg}$$

$$\text{Then } R = 4(D) = 3.92 \times 10^4 \text{ kg.}$$

Step 10

Check via the total equation

$$0.98 \times 10^4 + 1.02 \times 10^4 = 2.00 \times 10^4$$

Next let's start examining the general energy balance. You can reduce the number of variables involved by specifying that  $W = 0$  for the system and  $PE = KE = 0$  for each stream. The energy balance reduces to  $Q = \Delta H$ . The degrees-of-freedom analysis for the steady state system will be based on starting by considering every variable.

$$V = R + D = 4D + D = 5D = 4.90 \times 10^4 \text{ kg}$$

Assume  $V$  is in equilibrium with  $D$  at  $50^\circ\text{C}$ .

From Figure E24.8b the enthalpy data are

Variable	State	kg	Approximate $\Delta H(\text{kJ/kg})$
$F$	Saturated liquid, $\omega_{Bz} = 0.50$	20,000	165
$B$	Saturated liquid, $\omega_{Bz} = 0.04$	10,200	205
$D$	Saturated liquid, $50^\circ\text{C}$ , $\omega_{Bz} = 0.98$	9,800	100
$R$	Saturated liquid, $50^\circ\text{C}$ , $\omega_{Bz} = 0.98$	39,200	100
$V$	Saturated vapor, $\omega_{Bz} = 0.98$	49,000	540

An energy balance about the condenser will yield  $Q_C$ .

$$(39,200 + 9800)(100) - (49,000)(540) = Q_C = -2.38 \times 10^7 \text{ kJ (heat removed)}$$

An energy balance about the whole system is easier to use than a balance about the reboiler because the values of  $U$  and  $W$  are not involved.

Finally,

$$(9800)(100) + (10,200)(205) - (20,000)(165) = Q_R + Q_C = Q_R - 2.38 \times 10^7$$

$$Q_R = 2.36 \times 10^7 \text{ kJ}$$



Notice This

**Home Work:** *Basic Principles and Calculation in Chemical Engineering*, 7<sup>th</sup> edition, **Problems of chapter 24, page 749 – 762.**